New KPI Actuator Travel: Scaling and Aggregation

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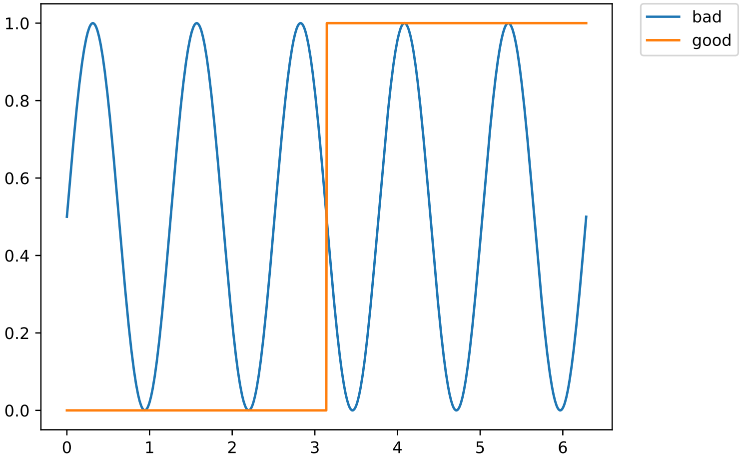
Motivation

A KPI that can measure hunting/ cycling:



Arclength

For a function $u(t)$, we can compute the arclength $L(u)$ as follows:

$$L(u; 0, T) = \int_0^T \sqrt{1 + (u'(t))^2} dt$$

There arise three issues:

- scaling/ comparing across different scales
- aggregating several actuators
- measure control signal or actual actuator travel?

Scaling/ comparing across different scales

How to compare actuators across different ranges $[u_{\min}, u_{\max}]$, $[0, T]$?

$[0, T]$:
Arclength $L(u) \Rightarrow$ Arclength ratio $L(u)/T$
(passage of time isn't controllers fault)

Scaling, advanced

Problem: Two trajectories that only have different indexing $[0, T]$ or actuator scale $[u_{\min}, u_{\max}]$ should have the same arclength-ratio.

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Base arclength ratio: 1.37
Scaled arclength ratio: 2.03
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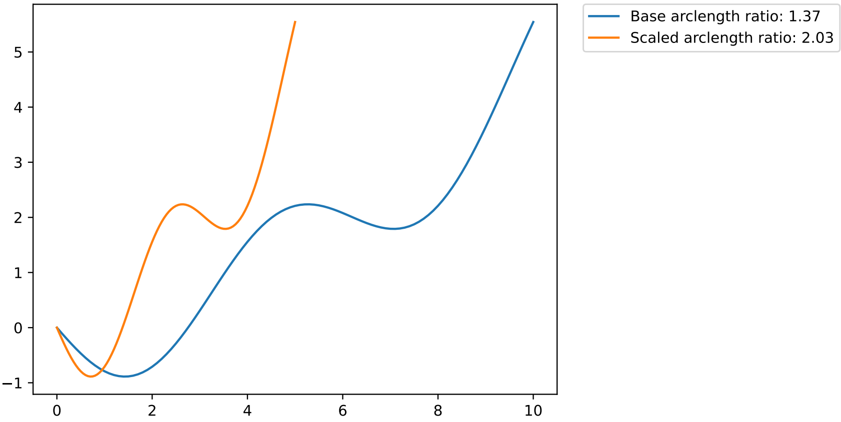
	y	scaled_index /2
0.0	0.000000	0.00
0.1	-0.099333	0.05
0.2	-0.196669	0.10
0.3	-0.291020	0.15
0.4	-0.381418	0.20

Scaling t changes ratio!



scaling t has to be dealt with carefully

$$\frac{1}{T}\int_0^T \sqrt{1+u'(t)^2}dt = \frac{1}{\alpha T}\int_0^{\frac{T}{\alpha}} \sqrt{1+u'(\alpha t)^2}dt$$
$$= \frac{1}{\alpha T}\int_0^{\frac{T}{\alpha}} \sqrt{1+\alpha^2 u'(t)^2}dt$$

...which can be proven by using the substitution rule for integration:

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(y)dy$$

TLDR on scaling

- u and t can't be scaled independently: scaling to $\frac{T}{\alpha}$ means also having to scale to αu (to keep arclength ratio the same)
- Compute arclength-ratio
  $\rightarrow$ comparison across time scales.
- Normalize actuator signals to $[0, 1]$
  $\rightarrow$ comparison across actuators (should be already the case?)

Aggregating several actuators $u_1, \ldots, u_n$

Suggestion (with u_i all already properly scaled):

$$KPI_{act} = \frac{1}{n} \sum_{i=1}^{n} \frac{L(u_i)}{T}$$

Then $KPI_{act} \in [1, \infty)$ for any n, where lower is better and 1 means no change in activation.

$\frac{1}{n}$ could be changed to more individual weights w_i (if certain actuators are more vulnerable to cycling).

Discussion

Issue 3:

- Measure control signal or what the actuator *actually* travels in the simulation?
  $\rightarrow$ We lean towards control signal, as the KPI is meant to measure bad control behavior, not actual wear-and-tear.